

B.Sc. Semester - 1 (CBCS) Examination
Oct/Nov. -2019 (NEW COURSE)
MATHEMATICS (CORE)

Time: 2:30 Hours

Marks: 70

Instructions:

1. All questions are compulsory.
 2. Figures to the right indicate marks.
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Q.1 (A) Answer any two of the following questions.**[10]**

- (1) State and prove Lagrange's mean value theorem and explain its geometrically.
- (2) For $0 < a < b$, show that $\frac{a^b - 1}{\log a} = a^c (1 + c^2) \tan^{-1} b$.
- (3) Find coefficient of x^3 in the expansion of $\log (e^x + 1)$ in ascending power of x .

Q.1 (B) Answer any one of the following questions.**[04]**

- (1) Verify Roll's theorem and find value of c for the function $f(x) = \frac{\sin x}{e^x}$, $\forall x \in [0, \pi]$.
- (2) Expand $\sin x$ in ascending power of $(x - \frac{\pi}{2})$.

Q.2 (A) Answer any two of the following questions.**[10]**

- (1) State and prove necessary and sufficient condition for a differential equation to be exact.
- (2) Define integrating factor and using that solve the differential equation $x^2 y dx - (x^3 + y^3) dy = 0$.
- (3) (a) Evaluate : $\lim_{x \rightarrow 1} \left(\frac{1}{\log x} - \frac{1}{x-1} \right)$,
 (b) Solve : $(y^2 + 2xy) dx - (x^2 + 2xy) dy = 0$.

Q.2 (B) Answer any one of the following questions.**[04]**

- (1) Solve : $y - \cos x \frac{dy}{dx} = y^2 (1 - \sin x) \cos x$.
- (2) Evaluate : $\lim_{x \rightarrow 0} \frac{(1 - \cos x)^{\frac{3}{2}}}{x - \sin x}$.

Q.3 (A) Answer any two of the following questions.**[10]**

- (1) Solve : $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$.
- (2) Solve : $y = 2px - (1/3)p^2$.
- (3) Solve : $\frac{d^4 y}{dx^4} + 8 \frac{d^2 y}{dx^2} + 16y = 0$.

Q.3 (B) Answer any one of the following questions.**[04]**

- (1) Solve : $(1 + y^2) dx = (\tan^{-1} y - x) dy$
- (2) Write general form of linear differential equation of n^{th} order, define its corresponding equation and complementary function.

Q.4 (A) Answer any two of the following questions.**[10]**

- (1) In usual notations prove that $\frac{1}{f(D)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V$, Where V is a function of x .
- (2) Solve : $(D^4 - 3D^2 + 2) y = \sin 4x$.
- (3) Solve : $(D^2 + 2D + 1) y = 2x + x^2$.

Q.4 (B) Answer any one of the following questions.**[04]**

- (1) In usual notations prove that $\frac{1}{f(D^2)} \sin ax = \frac{1}{f(-a^2)} \sin ax$.
- (2) Solve : $\frac{d^3y}{dx^3} - 12 \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} - 8y = e^{2x}$.

Q.5 (A) Answer any two of the following questions.**[10]**

- (1) Solve : $(x^2 D^3 - 4x D^2 + 6D) y = 4$.
- (2) Solve : $(x^2 D^2 - 2) y = x^2 + \frac{1}{x}$.
- (3) Write general form of homogeneous linear differential equation and explain a method of solving it.

Q.5 (B) Answer any one of the following questions.**[04]**

- (1) In usual notations prove that $\frac{1}{f(\theta)} x^m = \frac{1}{f(m)} x^m$.
- (2) Solve: $(x^2 D^2 - 2x D - 2) y = 0$
